DESCRIBING FUNCTIONS 1.1.3 through 1.2.2

In addition to introducing students to the classroom norms of problem-based learning, the main objective of these lessons is for students to be able to fully describe the key elements of the graph of a function. To fully describe the graph of a function, students should respond to these graph investigation questions:

Graph Investigation Question	Sample Summary Statement
What does the graph look like?	*The graph looks like half of a parabola on its side.*
Is the graph increasing or decreasing (reading left to right)?	*As x gets bigger, y gets bigger.*
What are the *x*- and *y*-intercepts?	*The graph touches both the x- and y-axes at 0.*
Are there any limitations on the inputs (domain) of the equation?	*Only positive values of x are possible. Zero is also possible.*
Are there any limitations on the outputs (range) of the equation? (Is there a maximum or minimum *y*-value?)	*The smallest y-value is 0. There is no maximum y-value.*
Are there any special points?	*The graph has a "starting" point at (0,0).*
Does the graph have any symmetry? If so, where?	*This graph has no symmetry.*

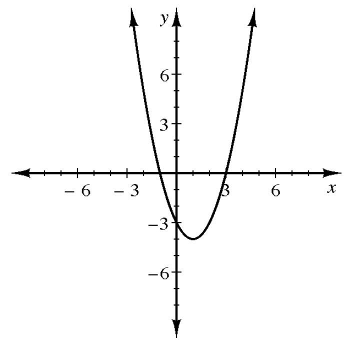
The more formal concepts of function and domain and range are addressed in Lessons 1.2.4 and 1.2.5.

For more information, see the Math Notes boxes in Lessons 1.1.1, 1.1.2, and 1.1.3. Student responses to the Learning Log in Lesson 1.1.1 (problem 1-32), if it was assigned, can also be helpful.

Example 1

For the equation $y = x^2 - 2x - 3$, make an $x \rightarrow y$ table, draw a graph, and fully describe the features.

At this point there is no way to know how many points are sufficient for the $x \rightarrow y$ table. Add more points as necessary until you are convinced of shape and location.



x	–2	–1	0	1	2	3
y	5	0	–3	–4	–3	0

Be careful with substitution and Order of Operations when calculating values. For example if $x = -2$, $y = (-2)^2 - 2(-2) - 3 = 5 = 4 + 4 - 3 = 5$.

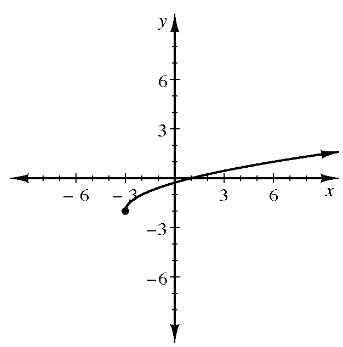
The graph is a parabola; it points upward. The x-intercepts are $(-1, 0)$ and $(3, 0)$. The y-intercept is $(0, -3)$. Reading from left to right, the graph decreases until $x = 1$ and then increases. The minimum (lowest) point on the graph (called the vertex) is $(1, -4)$. The vertical line $x = 1$ is a line of symmetry. There are no limitations on inputs to the function. Outputs can be any value greater than or equal to –4.

Example 2

For the equation $y = \sqrt{x+3} - 2$, make an $x \rightarrow y$ table, draw a graph, and describe the features.

Note that the smallest possible number for the $x \rightarrow y$ table is $x = -3$. Anything smaller will require the square root of a negative, which is not a real number.

x	–4	–3	0	1	3	6
y		–2	–0.3	0	0.4	1



The graph is half a parabola. It starts at $(-3, -2)$ and has x-intercept of $(1, 0)$ and y-intercept of $(0, \approx -0.3)$. The graph increases from left to right. The inputs are limited to values –3 or greater, and the outputs are limited to –2 or greater. There is no line of symmetry.

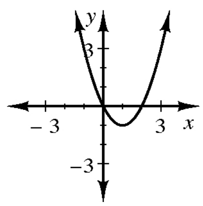
Problems

For each equation, make an $x \rightarrow y$ table, draw a graph, and describe the features.

1.	$y = x^2 - 2x$	2.	$y = x^2 + 2x - 3$	3.	$y = \sqrt{x-2}$
4.	$y = 4 - x^2$	5.	$y = x^2 + 2x + 1$	6.	$y = -\sqrt{x} + 3$
7.	$y = -x^2 + 2x - 1$	8.	$y = \lvert x+2 \rvert$	9.	$y = 2\sqrt[3]{x} - 1$

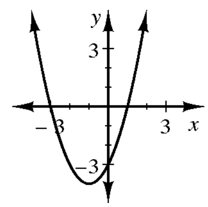
Answers

1.



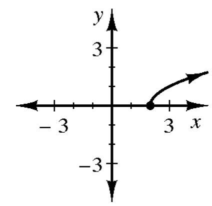
Parabola; intercepts (0, 0), (2,0); decreasing until $x = 1$ then increasing; minimum value at $(1, -1)$; $x = 1$ is a line of symmetry. Inputs can be any real number. Outputs are greater than or equal to –1.

2.



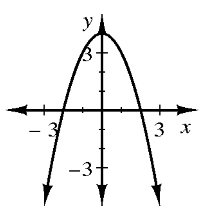
Parabola; intercepts (–3, 0), (1,0) and (0, –3); decreasing until $x = -1$, then increasing; minimum value at (–1, –4); $x = -1$ is a line of symmetry. Inputs can be any real number. Outputs are greater than or equal to –4.

3.



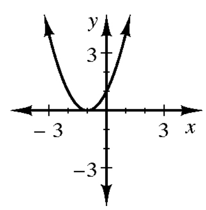
Half-parabola; starting point, intercept and minimum point (2, 0); increasing for $x > 2$. Inputs can be any number greater than or equal to 2. Outputs are greater than or equal to 0.

4.



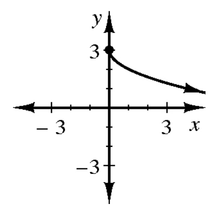
Parabola; intercepts (–2, 0), (2, 0) and (0, 4); increasing for $x < 0$, decreasing for $x > 0$; maximum value at (0, 4); $x = 0$ is a line of symmetry. Inputs can be any real number. Outputs are less than or equal to 4.

5.



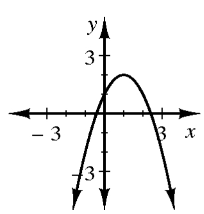
Parabola; intercept (–1, 0); decreasing for $x < -1$, increasing for $x > -1$; minimum value at (–1, 0); $x = -1$ is a line of symmetry. Inputs can be any real number. Outputs are greater than or equal to 0.

6.



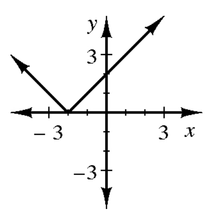
Half-parabola; starting point, intercept and maximum point (0, 3); decreasing for $x > 0$. Inputs can be any number greater than or equal to 0. Outputs are less than or equal to 3.

7.



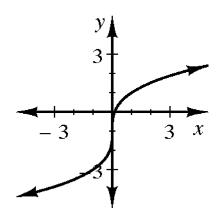
Parabola; intercepts (–0.4, 0), (2.4, 0) and (0, 1); increasing for $x < 1$, decreasing for $x > 1$; maximum value at (1, 2); $x = 1$ is a line of symmetry. Inputs can be any real number. Outputs are less than or equal to 2.

8.



V-shape; intercepts (–2, 0) and (0, 2); decreasing for $x < -2$, increasing for $x > -2$; minimum value at (–2, 0); $x = -2$ is a line of symmetry. Inputs can be any real number. Outputs are greater than or equal to 0.

9.



S-shape; intercept (0, 0); increasing for all x from left to right. Inputs and outputs can be any real number. There is no line of symmetry.

FUNCTIONS 1.2.4 through 1.2.5

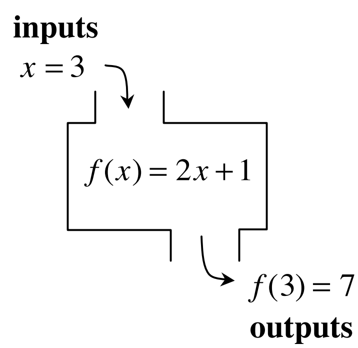
A relationship between the input values (usually x) and the output values (usually y) is called a function if for each input value, there is no more than one output value. Functions can be represented with an illustration of an input–output "machine," as shown in Lesson 1.2.3 of the textbook and in the diagram in Example 1 below.

The set of all possible inputs of a relation is called the **domain**, while the set of all possible outputs of a relation is called the **range**.

For additional information about functions, function notation and domain and range, see the Math Notes box in Lesson 1.2.5.

Example 1

Numbers, represented by a letter or symbol such as x, are input into the function machine labeled f one at a time, and then the function performs the operation on each input to determine each output, $f(x)$. For example, when $x = 3$ is put into the function f at right, the machine multiplies 3 by 2 and adds 1 to get the output, $f(x)$ which is 7. The notation $f(3) = 7$ shows that the function named f connects the input (3) with the output 7. This also means the point (3, 7) lies on the graph of the function.



Example 2

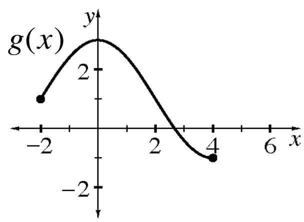
a. If $f(x) = \sqrt{x-2}$ then $f(11) = ?$ $\quad f(11) = \sqrt{11-2} = \sqrt{9} = 3$

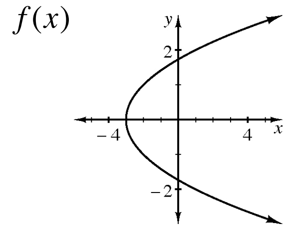
b. If $g(x) = 3 - x^2$ then $g(5) = ?$ $\quad g(5) = 3 - (5)^2 = 3 - 25 = -22$

c. If $f(x) = \dfrac{x+3}{2x-5}$ then $f(2) = ?$ $\quad f(2) = \dfrac{2+3}{2 \cdot 2 - 5} = \dfrac{5}{-1} = -5$

Example 3

A relation in which each input has only one output is called a **function**.





$g(x)$ is a function: each input (x) has only one output (y).
$g(-2) = 1$, $g(0) = 3$, $g(4) = -1$, and so on.

$f(x)$ is not a function: each input greater than -3 has two y-values associated with it.
$f(1) = 2$ and $f(1) = -2$.

Example 4

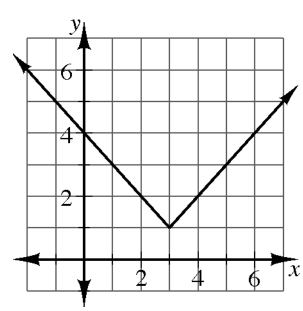
The set of all possible inputs of a relation is called the **domain**, while the set of all possible outputs of a relation is called the **range**.

In Example 3 above, the domain of $g(x)$ in Example 3 is $-2 \le x \le 4$, or "all numbers between -2 and 4." The range is $-1 \le x \le 3$ or "all numbers between -1 and 3."

The domain of $f(x)$ in Example 3 above is $x \ge -3$ or "any real number greater than or equal to 3," since the graph starts at -3 and continues forever to the right. Since the graph of $f(x)$ extends in both the positive and negative y directions forever, the range is "all real numbers."

Example 5

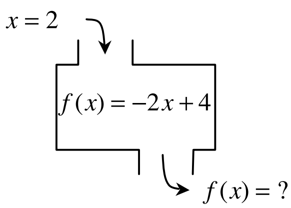
For the graph at right, since the x-values extend forever in both directions the domain is "all real numbers." The y-values start at 1 and go higher so the range is $y \ge 1$ or "all numbers greater or equal to 1."



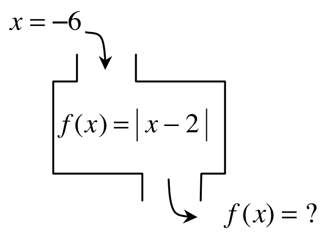
Problems

Determine the outputs for the following relations and the given inputs.

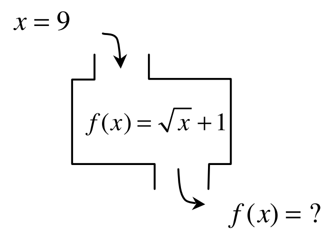
1.



2.



3.



4. $f(x) = (5 - x)^2$
   $f(8) = ?$

5. $g(x) = x^2 - 5$
   $g(-3) = ?$

6. $f(x) = \frac{2x+7}{x^2-9}$
   $f(3) = ?$

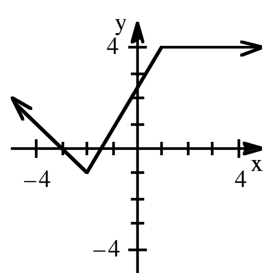
7. $h(x) = 5 - \sqrt{x}$
   $h(9) = ?$

8. $h(x) = \sqrt{5 - x}$
   $h(9) = ?$

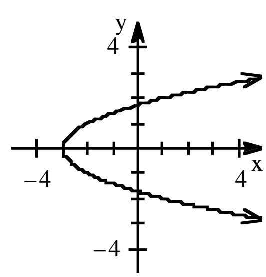
9. $f(x) = -x^2$
   $f(4) = ?$

Determine if each relation is a function. Then state its domain and range.

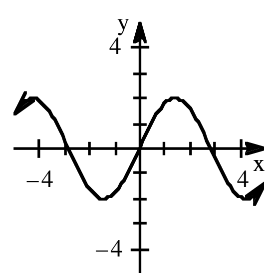
10.



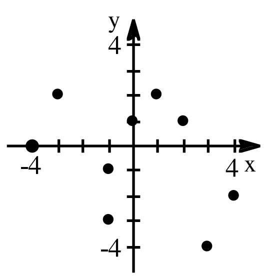
11.



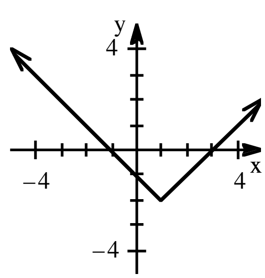
12.



13.



14.



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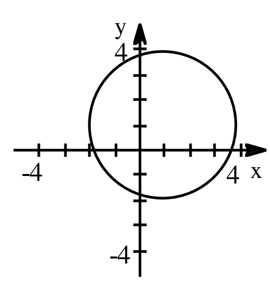

Answers

1. 0
2. 8
3. 4
4. 9
5. 4
6. not possible
7. 2
8. not possible
9. –16
10. yes, each input has one output; domain is all numbers, range is $-1 \le y \le 4$
11. no, for example x=0 has two outputs; domain is $x \ge -3$, range is all numbers
12. yes; domain all numbers, range is $-2 \le y \le 2$
13. no; -1 has two outputs; domain is -4,–3, –1, 0, 1, 2, 3, 4, range is –4, –3, –2, -1, 0, 1, 2
14. yes; domain is all numbers, range is $y \ge -2$
15. no, many inputs have two outputs; domain is $-2 \le x \le 4$ range is $-2 \le y \le 4$